Name: _____   **Math 254-1 Final Exam: 12/17/8**   ID Code: _____

1. Carefully define the term "vector space". Give two examples, neither of which may be $\mathbb{R}^n$ for any n.

2. Carefully define the term "independent". Give two examples in $\mathbb{R}^2$.

The following two questions involve the linear mapping $f : \mathbb{R}^3 \to \mathbb{R}^4$ given by $f(x, y, z) = (x - z, x - 2y, 2y - z, 6y - z - 2x)$.

3. Represent f as a matrix multiplication.

4. Determine the rank and nullity of f.

The following two questions involve the matrix $A = \left(\begin{smallmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \\ 1 & 0 & 2 \end{smallmatrix} \right)$.

5. Find A^{-1}, the inverse of A.

6. Use A^{-1} to find all solutions to the linear system $A[p \; q \; r]^T = [1 \; 2 \; 3]^T$.

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7. Find all solutions to the linear system $\{u - v + w = 1, \; u + 2v - 4w = 3, \; 2u + v - 3w = 4, \; 3u - 2w = 5\}$.

8. Consider the linear system $\left(\begin{smallmatrix} 1 & 2 \\ 3 & a \end{smallmatrix} \right) \left(\begin{smallmatrix} x \\ y \end{smallmatrix} \right) = \left(\begin{smallmatrix} -1 \\ -3 \end{smallmatrix} \right)$. For which values of a does this system have no solutions? For which values of a does this system have one solution? Infinitely many solutions?

9. Set $U = \{(a, b, c) : abc = 6; a, b, c \text{ are real}\}$, a subset of $\mathbb{R}^3$. Give three vectors from U, and determine whether or not U is a vector space.

10. Set $V = \mathbb{R}^5$. Give any two subspaces U_1, U_2 such that $U_1 \oplus U_2 = V$.

11. Let W be the vector space of functions that have as a basis $B = \{1, e^{2x}, \sin x, \cos x\}$. Let L be the linear operator on W that acts via $L(f(x)) = f''(x) - f'(x) + 2f(x)$. Find the representation $[L]_S$.

The following two questions involve the vector space $\mathbb{R}^3$ under the standard inner product, and the subspaces $S = \text{span}\{(1, 1, 2), (2, -1, 4)\}$ and $T = \text{span}\{(1, -2, 2)\}$.

12. Find an orthogonal basis for S.

13. Determine the dimensions of each of $S, T, S + T, S \cap T$.

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14. Find the determinant of $A = \left(\begin{smallmatrix} 1 & 1 & 0 & -1 \\ 0 & 2 & 3 & 1 \\ 2 & 1 & 0 & 2 \\ -1 & 2 & 2 & 0 \end{smallmatrix} \right)$.

The following two questions involve the matrix $A = \left(\begin{smallmatrix} 0 & 1 & 1 \\ -4 & 4 & 2 \\ 0 & 0 & 2 \end{smallmatrix} \right)$.

15. Find all of A's eigenvalues. Find a basis for each eigenspace.

16. For each eigenvalue, give the algebraic and geometric multiplicity. What is A's Jordan canonical form?

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